Soliton Solution, Bäcklund Transformation, and Conservation Laws for the Sasa-Satsuma Equation in the Optical Fiber Communications

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Under investigation in this paper, with symbolic computation, is the Sasa-Satsuma (SS) equation which can describe the propagation of ultra short pulses in optical fiber communications. By virtue of the Ablowitz-Kaup-Newell-Segur procedure, the Lax pair for the SS equation is directly established. Based on such a Lax pair, a Bäcklund transformation is constructed, through which the explicit one-soliton solution is derived. Meanwhile, an infinite number of conservation laws is provided to indicate the integrability of the SS equation in the Liouville sense. To further understand the stability of the one-soliton solution, we employ the split-step Fourier method to simulate the propagation of the soliton pulses under the finite initial perturbations. In addition, the interaction of two adjacent pulses with different separation distances is investigated through numerical simulation. Analytic and numerical results discussed in this paper are expected to be applied to the description of the optical pulse propagation.

Key words: Sasa-Satsuma Equation; Lax Pair; Bäcklund Transformation; One-Soliton Solution; Infinite Number of Conservation Laws; Soliton Interaction.
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